membrane stiffness contribution to frequency becomes high compared to the bending stiffness contribution. Similar arguments apply to the transverse shear model as bending stiffness is introduced into the model.

## References

[1] Beiner, L. and Librescu, L., "Minimum Weight Design of an Orthrotropic Shear Panel with Fixed Flutter Speed," *AIAA Journal*, Vol. 21, July 1983, pp. 1015-1016.

[2] Goland, M. and Luke, Y. L., "The Exact Solution for Two-Dimensional Linear Panel Flutter at Supersonic Speeds," *Journal of the Aeronautical Sciences*, Vol. 21, No. 4, 1954, pp. 275-276.

[3] Ashley, H. and Zartarian, G., "Piston Theory—A New Aerodynamic Tool for the Aeroelastician," *Journal of the Aeronautical Sciences*, Vol. 23, No. 12, 1956, pp. 1109-1118.

[4] Bolotin, V. V., *Nonconservative Problems of the Theory of Elastic Stability*, The Macmillan Co., New York, 1963, pp. 257-265.

[5] Ince, E. L., *Ordinary Differential Equations*, Dover Publications, New York, 1944, pp. 215-217.

# Reply by Authors to A. H. Flax

L. Beiner*
*Ben Gurion University, Beer Sheva, Israel*
and
L. Librescu†
*Tel Aviv University, Ramat Aviv, Israel*

BY developing an exact solution to Eq. (1) of Ref. 1 for the special case of uniform panel thickness, Dr. Flax points out that pure transverse shear panels (PTSP) cannot undergo flutter in the range of supersonic velocities for which piston theory applies. This is a paradoxical result, since it appears by considering, for example, the flutter of a sandwich-type panel consisting of two thin faces (modeled in accordance with Love-Kirchhoff theory), separated by a core modeled as a PTSP. As is well known, such a panel will flutter if the dynamic pressure becomes large enough. As the faces become thinner, the panel will flutter at lower dynamic pressures, i.e., become less stable. On the other hand, when in the limit the thickness of the faces vanishes (resulting in a structure with no bending stiffness), the flutter speed of the remaining PTSP becomes infinite according to Flax's exact solution. A paradox is thus reached in which a PTSP in high supersonic flow should be always stable, while more rigid structures (in bending) can flutter. This result belongs to the same class of aeroelastic paradoxes as the well-known membrane flutter paradox (see Refs. 2-4 of the Comment and our Refs. 2-4).

However, by using a singular perturbation method, a solution of the membrane flutter paradox has been developed in Ref. 5. This solution (which will be also useful for the problem at hand) considers the case of two-dimensional flat thin panels subjected to chordwise in-plane tensile stresses $\sigma_x$ and exposed over its upper face to a supersonic flowfield. It is assumed that $D \equiv E' h^3$ is the panel bending stiffness, where $E' \equiv E/12(1-\nu^2)$ denotes the reduced Young's modulus. Linear piston aerodynamics is employed. An "interior solution" is first constructed by removing the exponential factor $\exp(\alpha x)$ appearing in the exact membrane flutter solution (a similar factor appears in the exact PTSP solution of the Comment); subsequently, the bending stresses near the edges are accounted for by a "boundary-layer" approach, and by letting $D \to 0$, the following flutter criterion is obtained

$$\frac{\rho_\infty U_\infty^2}{M_\infty \sigma_x}\left(\frac{E'}{\sigma_x}\right)^{1/2} = \left(\frac{2}{3}\right)^{3/2} \tag{1}$$

The above flutter criterion for "zero-thickness plates"[5] was obtained as a zero-order solution in the perturbation process. In Ref. 6 the same result was obtained from Erickson's three-dimensional panel flutter solution by a limiting process $D \to 0$. Note from Eq. (1) that the geometrical and mechanical characteristics in the spanwise direction are not intervening. Having in view the similarity between the aeroelastic equilibrium equations of flat panels with in-plane tensile stresses and sandwich-type panels with PTSP core, a flutter criterion for uniform PTSP can be obtained from Eq. (1) by replacing $\sigma_x$ by $G_{13}$, thus becoming

$$(\Lambda_0)_* = \left(\frac{2}{3}\frac{G_{13}}{E'}\right)^{2/3} \tag{2}$$

where $\Lambda_0 = \kappa p_\infty M_\infty / E'$ defines a velocity parameter and $E'$ denotes the reduced Young's modulus of the faces whose thickness becomes zero in the limiting process. Therefore, finite flutter speeds are predicted by viewing the uniform PTSP as the core of a symmetrical sandwich structure with the thickness of the faces tending to zero.

In the numerical example presented in Ref. 1, the critical flutter speed $(\Lambda_0)_*$ of the uniform PTSP—which intervenes as a fixed parameter in the optimal solution—was obtained by a Galerkin technique, where the representation of $w$ corresponds to the interior solution of Ref. 5. It is of course advisable to use criterion (2)—believed by its authors[5] to be a crude but conservative one—for calculating the flutter speed of the uniform PTSP. It is hoped that the present discussion prompted by the Comment will stimulate the development of even more accurate solutions to the membrane and PTSP flutter problems by using higher order approximations in the perturbation method. This will also constitute better input data for the aeroelastic optimization problem considered in Ref. 1.

Received Nov. 14, 1983. 

*Associate Professor, Dept. of Mechanical Engineering.
†Professor, Dept. of Solid Mechanics, Materials and Structures.

## References

[1] Beiner, L. and Librescu, L., "Minimum Weight Design of an Orthotropic Shear Panel with Fixed Flutter Speed," *AIAA Journal*, Vol. 21, July 1983, pp. 1015-1016.

[2] Novitchkov, Yu. N., "Flutter of Plates and Shells," *Itogi Nauki i Techniki*, Vol. 11, 1978, pp. 67-122 (in Russian).

[3] Kornecki, A., "Aeroelastic and Hydroelastic Instabilities of Infinitely Long Plates II," *SM Archives*, Vol. 4, Issue 4, Nov. 1979, pp. 241-345.

[4] Ellen, C. H., "Approximate Solutions of the Membrane Flutter Problem," *AIAA Journal*, Vol. 3, June 1965, pp. 186-187.

[5] Spriggs, J. H., Messiter, A. F., and Anderson, W. J., "Membrane Flutter Paradox-An Explanation by Singular-Perturbation Methods," *AIAA Journal*, Vol. 7, Sept. 1969, pp. 1704-1709.

[6] Johns, D. J., "Supersonic Membrane Flutter," *AIAA Journal*, Vol. 9, May 1971, pp. 960-961.

# The Nondimensional Coefficient of Thermal Conductivity

C. M. Hung*
*NASA Ames Research Center*
*Moffett Field, California*

THE purpose of this Note is to draw attention to the misleading notation of the coefficient of thermal conductivity divided by Prandtl number, $\kappa Pr^{-1}$, in many

Received March 9, 1983; revision received Sept. 1, 1983. 

*Research Scientist, Computational Fluid Dynamics Branch. Member AIAA.